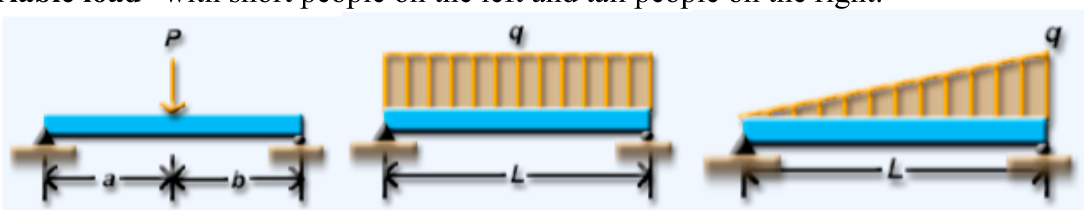
Beam Deflection Lab
General Information Worksheet

Your goal in this lab is to design a simply supported beam to hold a point load at its center with a specific deflection. Let's define some of these terms. You know what a **beam** is, they are used in the building you're in right now and even in your home, but they come with all kinds of cross sectional shapes. Most beams are named after what their cross section looks like, ex. an I-beam looks like an I. You can imagine what a T-beam, an L-beam, and Square-beam look like … you get the idea.

This figure shows several "**simply supported**" beams. This means that the ends just sit on their supports. The one on the left is a "**point load**" in the center. Imagine it's just one person standing in the center of the bridge. Compare it with a "**uniform load**" that might represent people standing all along the bridge, or the "**variable load**" with short people on the left and tall people on the right.



www.engineeringcalculator.net

How much these beams deflect, bend, or deform (all interchangeable terms) is based on several variables. What criteria do you think affects the amount a beam deflects?

A. The material the beam is made from.
B. ___________________
C. ___________________
D. ___________________
E. ___________________

In our "simplified" discussion in class we're going to limit this list (there are lots of variables which affect deflection). We'll start with two primary elements. The first is the material the beam is made out of. You intuitively know that a metal bridge is stronger than a wooden bridge. That's an affect of the material; different materials have different characteristics when stressed (like when they're bent). We're going to use **Young's Modulus** (sometimes called the **Modulus of Elasticity**) to take this characteristic into account. It's represented by the capital letter **E**, and has **units of pressure** (psi or N/m^2). Values for E are found in tables like this one.

Material	Young's Modulus (Modulus of Elasticity)	
	(10^6 psi)	(10^6 N/m^2)
ABS plastics		2.3
Aluminum	10.0	69
Bone		9
Carbon Fiber Reinforced Plastic		150
Chromium	36	
Cobalt	30	

The other criteria which heavily affects beam bending is the *cross sectional shape* of the beam we discussed above (I-beam vs. a Square-beam). This "shape" effect is quantified by what is called the **Moment of Inertia, *I***. For example, you intuitively know that a ruler bends easier when it's flat than when it's vertical. This may be a difficult concept to fully understand at first, but you'll understand it much better by the end of this lab. So a beam's moment of inertia, I, is going to use beam geometry (height and width) to help us determine how much that beam shape tends to bend.

Worksheet #1: Young's Modulus of Elasticity

Finding Young's Modulus of Your Material

The modulus of elasticity, **E** (more specifically called **Young's modulus**), is a measure of the stiffness of an elastic material and is a quantity used to characterize materials. Different materials (like steel and aluminum) have different modulus', many of which are listed in tables or other references. However, these values may not accurately represent your specific material, especially if it's an organic material (like wood). So, your first objective is to find E for your material.

We know the following two equations as they relate to beams with a rectangular cross section.

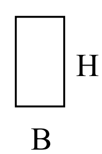
Eqn. 1 $y_{\max} = \dfrac{W \cdot L^3}{48 \cdot E \cdot I}$ Solve for $E =$ ________________________ **: Eqn. 2**

Eqn. 3 $I_{rect'} = \dfrac{B \cdot H^3}{12}$ Moment of Inertia (**shape dependent**)

Substitute Eqn. 3 into Eqn. 2 $E =$ ___________________________ **: Eqn. 4**

So, Eqn. 4 is based on the following variables. Take the rectangular beam you've been given and fill in the table below with measurements. (show the units too!) Use a load that will make a deformation you can easily and accurately measure, and a span that is almost as long as the beam.

The load in the center of the beam	$W =$	
The length of the span	$L =$	
The dimension of the cross section of your rectangular beam, base & height	$B =$	
	$H =$	
The maximum deflection at midpoint	$y_{\max} =$	



All of this allows you to find E for your material by substituting these values above into Eqn. 4.

Calculate Young's Modulus for your material. $E =$ ________(W for one roll of pennies)

$E =$ ________(W for two rolls of pennies)

Rotate your beam 90º on edge (B = 1/16", H = 3") and calculate y_{max} using the avg. of E above

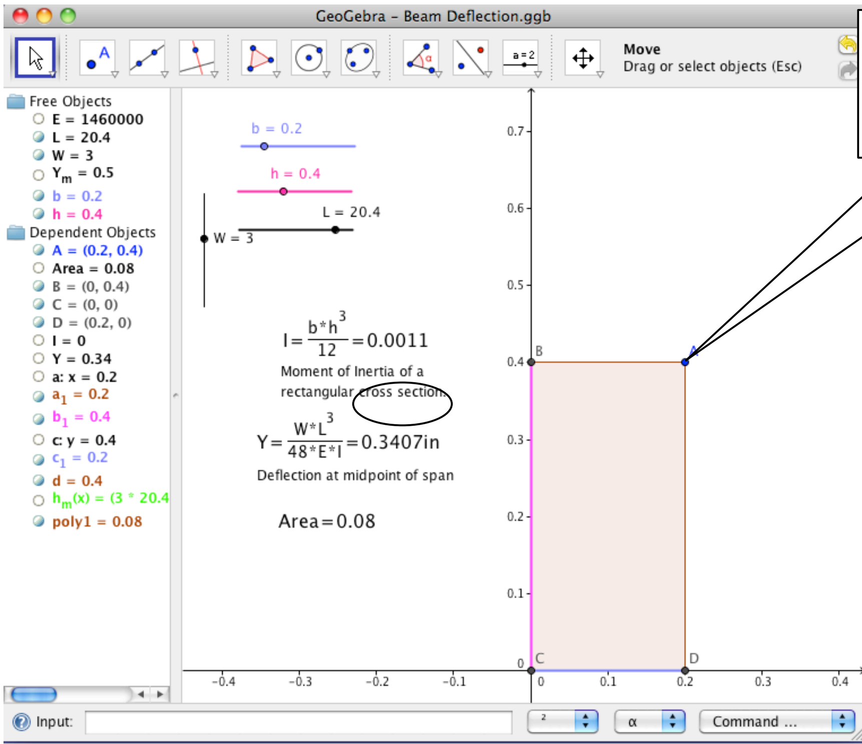
$y_{\max} =$ ________(W for one roll of pennies)

$y_{\max} =$ ________(W for two rolls of pennies)

Optimizing for Cost

In this exercise we're going to investigate optimizing beam performance. In "real life," project costs increase as materials increase, and cross section will reflect how much material is used.

Open the GeoGebra file, Beam_Deflection.ggb. On the top left are sliders that will manipulate the length of the base (b), height (h), beam length (L), and load weight (W). The software calculates the moment of inertia I, and the maximum deflection Y_m. (Units are inches and pounds).



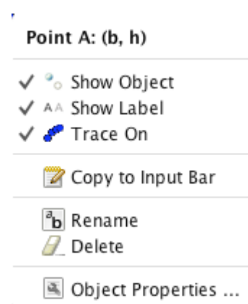
Grab and drag point A until the deflection is 0.50"

What are the coordinates of A(b, h)

____

What is the cross-sectional area?

____



Is this the smallest cross sectional area with a deflection of 0.25 inches? Right-click on point A and toggle "Trace On". Continue to manipulate point A to find other points which meet the deflection criteria.

What is the smallest value of cross sectional Area you can get? What are the values of b and h?

Area= ___________   when (b, h) is (_____ , ______)

Is this a reasonable shape for a beam? Why or why not?

Toggle on the equation $h_m(x)$ by clicking on the dot.
What might this represent?

Where did it come from? i.e. what is the formula for this curve and how do you find it?

Final Project
Building a Bridge

You now have the basic knowledge of beam deformation. Your task is to use that information to build a bridge meeting the following set of criteria.

Materials:

You will be supplied with two 1/16" x 3" x 24" pieces of basswood and wood glue. You will have access to a measuring device, a steel straight edge, and a utility knife, all of which you can use to cut up your two pieces of basswood into a bridge.

Criteria:

- The finished product must
    - Span 20"
    - Deflect 1/4" when given a 20 lb. point load at the center of the span
    - Be stable enough to accept the load without added help from you
    - Utilize the minimum amount of material.
    - (this is not a "glue" project, so bridges which attempt to use the glue as a supporting member will be disqualified)
- One piece of basswood will be your roadbed. <u>You cannot cut this piece</u>.
- The remaining piece will be used as support structure.
    - Support beams should be uniform and extend the full length of the bridge (so you can calculate the moment of inertia)

Order of Events:

Use a team notebook to show all of the following work. Be detailed and show calculations, equations, units, etc. to document your work. Do not erase anything in this notebook, it is a record of your efforts and will be used to double-check your work.

1. With your partners, take 10 minutes to sketch out as many ideas as you can.
2. Review your ideas and narrow them to those that you think will succeed.
3. Calculate the moment of inertia, I for your cross section. Use all resources at your disposal to do this; see list below for additional resources.
    i. Show all drawings with measurements, list your variables, and show equations.
    ii. Use I to calculate your deflections (goal is 0.25").
    iii. Make adjustments to optimize your design by minimizing the cross sectional area. This is a good place for a computer to do a lot of work for you, ex. set up a spread sheet to automatically make adjustments.
    iv. Recalculate I. Repeat till you have a good design plan. Put your plan and iterations into your notebook.
4. Take measurements and decide how you plan to cut and glue your pieces. Put this in your notebook.
5. Carefully glue up your pieces. It's tricky to keep things square and flat—BE PATIENT! You only have one shot!

Resources for finding the moment of inertia of composite shapes.

- http://www.nilit.com/plastics/images/ts/Inertia.gif

- http://www.engineersedge.com/section_properties_menu.shtml
- http://www.wolframalpha.com/input/?i=+beam+moment+of+inertia
- http://www.wolframalpha.com/input/?i=+beam+deformation

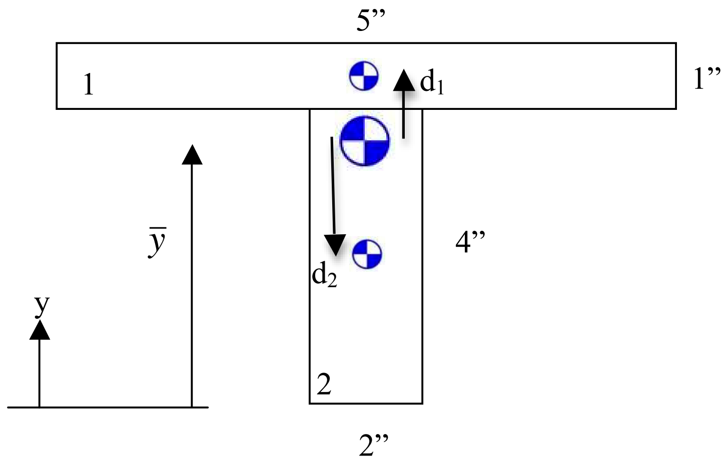
Finding I for Odd Shapes
(Composite Cross Sections)

While coming up with ideas for your bridge support you may come up with some cross sections which you do not know how to calculate the value for the moment of inertia, I. Here is a quick discussion of how you can calculate I for just about any combination of rectangular cross sections.

1. Break your shape into rectangles and number them. In this example there are only two but you could easily have more.

2. There are two calculations necessary to find I.
   a. Find the new centroid, $\bar{y}$ of the cross section (where shape balances)
   b. Use that to find the composite moment of inertia.

Here are the formulas.



Eqn 1. $\bar{y} = \dfrac{\sum A_i y_i}{\sum A_i}$

Eqn 2. $I = \sum\left[I_i + A_i d_i^2\right]$

In Eqn. 1 y_i is the distance from the baseline up to the center of mass of each individual piece (small dots). We can expand Eqn. 1 for our example to be

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(1"\times 5")(4"+0.5") + (2"\times 4")(2")}{(1"\times 5") + (2"\times 4")}$$

$$\bar{y} = 2.96$$

This is the centroid of the cross section (big dot) and will be used to find d which is the distance from $\bar{y}$ to the centroid of each individual area. Remember that for a rectangle, $I_{rect'} = \dfrac{B\cdot H^3}{12}$. So, from Eqn. 2

Eqn 3. $I = \sum\left[I_i + A_i d_i^2\right] = \left[I_1 + A_1 d_1^2\right] + \left[I_2 + A_2 d_2^2\right]$

Calculate I_1 and I_2 separately as well as d_1 and d_2.

$I_1 = \dfrac{B\cdot H^3}{12} = \dfrac{5\cdot 1^3}{12} = 0.417 in^4$ $\qquad$ $I_2 = \dfrac{B\cdot H^3}{12} = \dfrac{2\cdot 4^3}{12} = 10.7 in^4$

(note how much greater I_2 is! Why?)

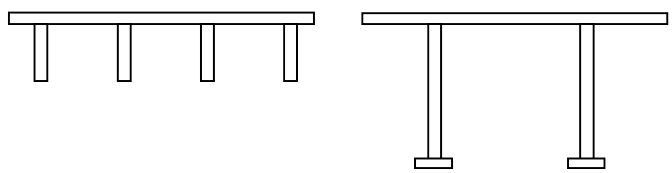
$d_1 = \bar{y} - 4.5 = 2.96 - 4.5$ $\qquad$ $d_2 = \bar{y} - 2 = 2.96 - 2$

$d_1 = -1.54 in$ $\qquad$ $d_2 = 0.960 in$

We can now put this into Eqn. 3 (Note: signs are going to be canceled out when it's squared)

$$I = \sum\left[I_i + A_i d_i^2\right] = \left[I_1 + A_1 d_1^2\right] + \left[I_2 + A_2 d_2^2\right] = \left[0.417 + 5\cdot(-1.54)^2\right] + \left[10.7 + 8\cdot 0.96^2\right]$$
$$I = 30.35 in^4$$

Having done all that … you'll be frustrated to know that there is already an equation for finding I for a T-beam! Simple shapes like this are listed in tables you can find on the web. **However**, even a small change like the figures below are not listed in tables, which means you will have to use this process to calculate I. So, learning this process will be useful as you engineer an awesome solution to this problem! p.s. consider using spreadsheets to crunch the numbers—they're really good at it and if you feed it the right variables & equations *it will allow you to make changes in a flash* as you proceed through the engineering process.



Instructor Resources

There is a lot of material on beam deformation on the web (and in textbooks, of course). Here is a collection you might find helpful.

Video instruction.

- http://youtu.be/4FBaa82r_7A (2 videos for intro, and more for related subjects. Easy to understand instruction)
- http://youtu.be/Qi8k6RjjhK4 (from the same author, this on finding I of a composite cross section)
- http://www.youtube.com/watch?v=asBW0Ojc0bY (5th video in a set of 10. Good graphics)

Beam deformation calculators

- http://www.wolframalpha.com/input/?i=+beam+deformation
- http://www.engineeringcalculator.net/beam_calculator.html

Moment of inertia formulas and/or calculators

- http://www.engineersedge.com/section_properties_menu.shtml
- http://www.wolframalpha.com/input/?i=+beam+moment+of+inertia
- http://www.nilit.com/plastics/images/ts/Inertia.gif

Fun and games that demonstrate bridge building

- http://bridgecontest.usma.edu/ This free bridge building simulator is from the US Military Academy and covers cost as well as design.
- http://www.physicsgames.net/game/Cargo_Bridge.html This is a more animated version of bridge building.